

# Damages Regimes, Precaution Incentives, and the Intensity Principle

by

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This paper revisits the accident model at its roots and shows that the intensity principle provides a powerful analytical tool to handle a variety of issues in a unifying frame and based on common intuition. If courts impose inefficient standards, if a cap on liability exists, or if the principal must pay an information rent to induce precaution, the exact method of quantifying damages matters. The intensity principle allows comparing the intensity of precaution incentives under different damages regimes, such as strict liability, proportional liability, and the negligence rule. Moreover, it requires less restrictive assumptions than the more traditional approach. (JEL: K13, D62)

## 1 Introduction

For applications of microeconomic theory, assumptions such as differentiability and convexity are commonly imposed to make use of calculus, the implicit-function theorem, and the first-order approach. Yet this comes at a price, as applications do not always fit into a setting based on a continuum of real numbers – or, as Milgrom and Shannon (1994) have phrased it, such assumptions play the role as servants to a method.

The accident model as the workhorse of the economic analysis of tort law is a prime example. Think of an accident due to an excavation pit. Under a negligence regime, courts will check whether or not the injurer has posted sufficient warning signs, whether or not he has illuminated these signs, whether or not he has built a fence around the pit, whether or not he has covered the pit over weekends, and so on. Such cases are easily captured by a finite set of available precaution measures

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(including combinations of them), initially without any formal structure. Continuous choice, as required by the first-order approach, may be more demanding to justify.

For finite sets, the method of calculus does not apply, but Milgrom and Shannon (1994) have provided a powerful substitute that is simple to handle. The present paper refers to the substitute technique as the *intensity principle*, because comparing the intensity of precaution incentives under different damages regimes is involved.

To illustrate and to propagate the method, precaution incentives are compared for three damages regimes – strict liability, proportional liability, and the traditional negligence rule – as well as in relation to first best. Both the negligence rule and proportional liability are based on due-care standards. While the negligence rule adheres to an all-or-nothing compensation of the victim, proportional liability aims at capturing just that part of the loss that is actually caused by the injurer's deviation from due precaution.

The analysis will be comprehensive in that arbitrary due-care standards are combined with limited liability and wealth constraints. Moreover, the injurer may be an agent who takes precaution on behalf of his principal. Vicarious liability can be viewed as weakening the agent's wealth constraint by combining the agent's assets with those of the principal. But, due to moral hazard or adverse selection, the interaction of principal and injurer under vicarious liability may be distorted. In particular, the principal may have to leave a rent to her agent in excess of the agent's outside option. The present paper covers all these aspects in the unifying frame and intuition of the intensity principle, which rests on the purely ordinal concept of monotonicity. Convexity of rents, as would be required under the first-order approach, is not needed by an analysis based on the intensity principle.

New results emerge, but some of my findings are also reminiscent of known results. In fact, Kahan's (1989) comparison of the negligence rule with his own rule (which is closely related to proportional liability) is contained as a special case of the present findings. Stremitzer and Tabbach (2009) have compared precaution incentives in a setting of limited liability to show that proportional liability outperforms the other damages regimes. The present paper offers a more general version of their findings and points out limits of the result. Proportional liability loses its superiority whenever a principal vicariously held liable must pay an information rent to induce precaution by her agent.

The traditional literature on vicarious liability has focused on the effect of increasing the ability to cover damages claims by adding the principal's assets. Important insights have been provided by several contributions. Kornhauser (1982) and Sykes (1984) examine wealth constraints of the agent as the basic condition favoring vicarious liability. Again, findings of the present paper based on the intensity principle can be seen as generalizing some of theirs.

Demougin and Fluet (1999) have compared strict liability with the negligence rule for vicarious liability where the principal-agent relationship is plagued by moral hazard and adverse selection. The present paper adds proportional liability. Moreover, in the adverse-selection case, Demougin and Fluet have courts imposing the ex post efficient precaution expenditures as due-care standards, while the present

paper allows for a wider variety of standards, including those that are not type-contingent and thus may come closer to what courts actually do impose.

Under both moral hazard and adverse selection, the principal is confined to precaution levels that are implementable by appropriate contracts. The sets of implementable precaution levels are endogenous and may contain holes even if the set of feasible care levels consists of all nonnegative real numbers. Applying calculus to sets with holes becomes tedious. The intensity principle, in contrast, does not face any comparable difficulties.

The paper is organized as follows. In section 2, a version of the accident model is introduced that distinguishes between precaution measures, precaution costs, and the accident probability. Some precaution measures can be ruled out, as they are dominated (from the injurer's perspective) by other measures. Rational injurers will never choose dominated precaution measures. Ruling dominated measures out leaves a set of actions that can be ordered in accordance with the level of precaution spending.

Section 3 compares the intensity of precaution incentives under three damages regimes: the negligence rule, proportional liability, and strict liability. There may be a cap on damages awards, which either reflects legally imposed limited liability or the wealth constraint of the injurer. The intensity principle is established, which shows that one regime provides more intensive precaution incentives than another one if the difference of the injurer's objective functions under the two regimes is monotonically increasing. At a fixed due-care level, proportional liability leads more closely to the first-best solution than does the negligence rule.

In substance, the findings of this section are known. Establishing them again by making use of the intensity principle serves two purposes. First, it illustrates the intensity principle at work in a surrounding familiar to many readers of this paper. Second, it prepares for the insight gained in later sections that the logic and intuition of the intensity principle easily extends to precaution decisions reached by principal-agent relationships. The first-order approach, in contrast, would require new proofs.

The next section, in fact, considers an agent taking precaution measures on behalf of a principal. In case of an accident, the principal may be held vicariously liable. Due to the added wealth of the principal, vicarious liability can be interpreted as raising the cap. In the presence of moral hazard, however, the interaction between principal and agent may be distorted. To induce sufficient precaution, the principal must pay to her agent an information rent increasing with precaution. In the intensity of precaution incentives, not much changes. The information rent simply cancels as far as the difference of the objective function under different regimes is concerned.

The superiority of proportional liability on efficiency grounds, however, may be lost because the rent, which is of purely redistributive nature, does not affect welfare. Yet, the wider range of precaution measures implementable under the negligence rule allows outperforming proportional liability even if the relationship between principal and agent suffers from moral hazard.

The next section investigates a setting where the principal–agent relationship is governed by adverse selection. Compared with previous sections, quite restrictive assumptions of the single-crossing type are needed to keep the principal–agent problem tractable. Monotonicity of rents, however, emerges directly from these incentive constraints. Convexity of rents, as would be needed under the first-order approach, can be dispensed with by relying on the intensity principle instead. Moreover, the same intuition as under moral hazard supports the findings, because rents cancel out whenever differences of the principal’s objective function under different regimes are under consideration.

Insights from mechanism design are needed for either approach. To set off the issues, I have relegated them to the appendix.

## 2 The Accident Model

The traditional accident model specifies the probability  $\varepsilon(x)$  of an accident as a monotonically decreasing function of precaution expenditures  $x$  from the set  $\mathbb{R}_+$  of nonnegative real numbers. If an accident occurs, the victim suffers a loss of fixed size  $L$ , and, depending on the damages regime in place and possibly the precaution expenditures actually made, the injurer may owe damages to the victim.

Under strict liability  $S$ , if an accident has occurred, damages  $D_S = L$  are due, no matter what precaution the injurer has actually taken. The other two rules considered by the present paper are based on a due-care standard  $x^o$ . Suppose the injurer has actually spent  $x$ . Then the negligence rule awards damages  $D_N(x, x^o) = L$  equal to the total loss if  $x < x^o$  but  $D_N(x, x^o) = 0$  otherwise, whereas proportional liability awards

$$D_P(x, x^o) = \frac{\varepsilon(x) - \varepsilon(x^o)}{\varepsilon(x)} \cdot L$$

if the injurer is found negligent ( $x < x^o$ ) and  $D_P(x, x^o) = 0$  otherwise. Proportional liability is in the spirit of Kahan’s (1989) rule, as it attempts to award only that part of the loss that is caused by the deviation from the standard. Shavell (1985) has investigated proportional liability to exempt the injurer if the loss has actually been caused by nature (but-for test).

In legal practice, courts are unlikely to focus solely on the actual amount  $x$  spent on precaution. Think of the example from the introduction, where courts would check whether or not the injurer has posted a warning sign and whether or not he has built a protective fence around the pit. Or think of a car accident. The driver may or may not have obeyed the speed limit. The tires of his car may or may not have been in proper condition. The brakes may or may not have been well maintained, and so on. The set of all conceivable precaution measures (including combinations of them) is denoted by  $A$ . This set may be envisaged as a subset of the  $n$ -dimensional real vector space, i.e.,  $A \subset \mathbb{R}^n$ , where some dimensions capture binary decisions while others involve continuous choice. It could even be appropriate to think of  $A$  as a finite set without much formal structure.

In any case, precaution measures impose costs to be borne by the injurer. Let  $c(a)$  denote the costs (expressed in monetary equivalent) associated with the precaution measure  $a \in A$ . These costs may reflect disutility of effort or, alternatively, may correspond to precaution expenditures. The probability  $p(a)$  of an accident also depends on the precaution measure  $a$  that the injurer has taken ex ante.

In this setting, it might be more appropriate to have courts focusing on the accident probability  $p(a)$  than on the costs  $c(a)$  actually incurred. A tiny fence offering little protection will not impress courts just because it was made out of unreasonably expensive material. For this reason, I rather specify a due accident probability  $p^o$  such that a precaution action  $a$  is negligent if it causes an accident with a probability  $p(a)$  higher than this standard, i.e.,  $p(a) > p^o$ .

In the same spirit, let me assume that damages awarded would not increase if a precaution measure were chosen with a lower accident probability. It then follows that the damages  $d(p(a), p^o) \geq 0$  due in case of an accident are a function of the actual accident probability  $p(a)$  and the probability standard  $p^o$ . In any case, I assume that damages due in case of an accident are a (weakly or strictly) decreasing function of the accident probability that comes with the chosen precaution measure, i.e.,

$$d(p(a), p^o) \geq d(p(a'), p^o)$$

is assumed to hold for any due accident probability  $p^o$  and for any two measures  $a$  and  $a'$  from  $A$  with  $p(a) \geq p(a')$ . Notice that, as a degenerate case, this condition would also be met in the case of strict liability, where the damages  $d(p(a), p^o) = d(p(a'), p^o) = L$  are independent of the true and the due accident probability.

In this setting, it proves useful to introduce the following notion of dominance between precaution measures. Measure  $a$  is said to dominate measure  $a'$  if one of the following two conditions is met. First, the two measures  $a$  and  $a'$  induce the same accident probability  $p(a) = p(a')$  but  $a'$  causes higher costs, i.e.,  $c(a') > c(a)$ . Second, measure  $a$  comes with a strictly lower accident probability (i.e.,  $p(a) < p(a')$ ) but does not cost more than  $a'$  (i.e.,  $c(a) \leq c(a')$ ). Let  $A^u$  denote the set of undominated measures that are left after all dominated measures have been eliminated.

Notice that this elimination procedure does not depend on the damages regime in place. Yet, as long as damages due in case of an accident are a (weakly or strictly) decreasing function of the accident probability, the rational injurer will not take any precaution measure that is dominated by another one in the above sense.

In fact, the (risk-neutral) injurer maximizes an objective function of the form

$$\phi(a, p^o) = G - c(a) - p(a) \cdot d(p(a), p^o),$$

where  $G$  captures the injurer's private gain from running the activity. Therefore, if measure  $a$  dominates  $a'$  in the sense of the first condition, it follows that

$$\begin{aligned} \phi(a, p^o) &= G - c(a) - p(a) \cdot d(p(a), p^o) \\ &> G - c(a') - p(a) \cdot d(p(a), p^o) = \phi(a', p^o), \end{aligned}$$

and hence the expected payoff is lower under  $a'$  than under  $a$ . Similarly, if  $a$  dominates  $a'$  in the sense of the second condition, then at least

$$\phi(a, p^o) \geq \phi(a', p^o)$$

must be true. Notice that this inequality holds in the strict sense again if either  $c(a) < c(a')$  or  $d(p(a), p^o) < d(p(a'), p^o)$ .

In any case, the injurer's payoff under a measure  $a'$  that is dominated by  $a$  cannot exceed the one under measure  $a$ , and hence he at least weakly (if not strictly) prefers measure  $a$  over measure  $a'$ . Notice that this claim is valid for any legal regime awarding damages that are a (weakly or strictly) decreasing function of the accident probability and for any due accident probability, efficient or not.

Therefore, if I no longer take dominated precaution measures into account, it is not because some invisible hand has removed them, but because an endogenous party, namely the injurer, never has the incentive to choose them.

The crucial feature of the set  $A^u$  of undominated precaution measures rests on the fact that its complete ordering based on the accident probability is equivalent to the one based on precaution expenditures. More precisely, for any two measures  $a$  and  $a'$ , both from the set  $A^u$  of undominated precaution measures,  $p(a) < p(a')$  holds if and only if  $c(a) > c(a')$ , and  $p(a) = p(a')$  holds if and only if  $c(a) = c(a')$ . From a formal perspective, specifying a due accident probability becomes equivalent to specifying a due level of spending. But notice: such a claim is valid only after dominated precaution measures have been eliminated.

Let

$$X = \{x \in \mathbb{R}_+ : \exists a \in A^u \text{ such that } x = c(a)\}$$

denote the set of precaution spendings arising from undominated precaution measures. Associated with any given level  $x$  of spending from  $X$ , an accident probability

$$\varepsilon(x) = \min_{a \in A} p(a) \quad \text{subject to} \quad c(a) \leq x$$

can unambiguously be defined. Notice that any undominated precaution measure  $a$  for which  $c(a) = x$  holds solves the above problem, i.e.,  $\varepsilon(x) = p(a)$ .

Moreover, by construction of the set  $X$ , the probability  $\varepsilon(x)$  is a strongly monotonically decreasing function of precaution spendings  $x$  from  $X$ .

If the idea that courts focus on precaution measures, not just on spending, is taken seriously, then the set  $X$  of levels of spending arising from undominated precaution measures must be derived from the primitives of the model. Since the set  $A$  of all precaution measures has little formal structure, the set  $X$  of spending levels arising from undominated precaution measures may also have little formal structure except for being a (possibly strict) subset of real numbers. Fortunately, for the method propagated by the present paper, the exact shape of the set  $X$  does not matter.

A more subtle point remains to be settled. If courts impose a due accident probability  $p^o$ , it cannot be taken for granted that there exists an undominated precaution measure  $a^o \in A^u$  that comes with an accident probability  $p(a^o)$  exactly equal to the due care probability, i.e.,  $p(a^o) = p^o$ . Yet, to avoid lengthy case distinctions, let me simply assume that courts voluntarily impose due accident probabilities that

injurers can exactly meet with some undominated precaution measure. This means that a due-care level  $x^o \in X$  is assumed to exist, such that  $\varepsilon(x^o) = p^o$  exactly holds.

Except for  $X$  possibly not being a continuum, we are back to the setting of the traditional accident model if dominated precaution measures are ruled out. The accident probability emerges as a monotonically decreasing function of precaution costs by construction. Convexity (and differentiability), however, as would be needed for the first-order approach, remains more demanding to justify.

### 3 Comparing Precaution Incentives

In the following, precaution incentives will be compared under strict liability  $S$ , the negligence rule  $N$ , and proportional liability  $P$  as introduced at the beginning of the previous section. The injurer chooses a precaution spending from the set  $X$  that captures spending levels arising from undominated precaution measures. The accident probability  $\varepsilon(x)$  is a strongly monotonically decreasing function of such spendings.

If the injurer has sufficient wealth to cover damages claims and if the standard is specified at its efficient level, then all three regimes provide efficient precaution incentives. If, however, standards are inefficient or if there is a cap  $H \leq L$  on liability, the three rules provide precaution incentives of different intensities.

Think of  $H$  as a legally imposed cap on liability. Alternatively, the cap may be due to the injurer's wealth constraint. If precaution costs are of the type of disutility of effort, the cap will remain constant. If, however, precaution expenditures have to be covered from the injurer's assets, the cap will decrease with increasing precaution spending (unless an upper bound on liability is set by the legislator, in which case it will remain constant again). To begin with, I assume a constant cap. At the end of the present section, the analysis is extended to nonconstant caps.

Under strict liability, the injurer's objective function is  $\phi_S(x) = G - x - \varepsilon(x) \cdot H$ , whereas under the other two rules  $J \in \{N, P\}$  based on a due-care standard  $x^o$ , his objective function is

$$\phi_J(x, x^o) = G - x - \varepsilon(x) \cdot \min[D_J(x, x^o), H].$$

Notice that, under the negligence rule as well as under proportional liability, the injurer will never choose precaution in excess of the standard, that is, his choice will be from the range  $I(x^o) = \{x \in X : x \leq x^o\}$ . His optimal choice need not be unique. Let

$$M(J, x^o) = \arg \max_{x \in I(x^o)} \phi_J(x, x^o)$$

denote the set of all precaution costs that maximize the corresponding objective function.

The injurer under strict liability, as well as the social planner with expected welfare  $\phi_W(x) = G^s - x - \varepsilon(x) \cdot L$  as objective function, may choose precaution in excess of the standard ( $G^s$  denotes the social gain from running the activity, which

may differ from the injurer's private gain  $G$ ). Yet, it proves analytically convenient to consider the set of precaution costs

$$M(J, x^o) = \arg \max_{x \in I(x^o)} \phi_J(x)$$

that maximize these objective functions artificially constrained to the range  $I(x^o)$  nonetheless. For regimes  $J \in \{S, W\}$ , the actual choice would be from  $M(J, \infty)$ .

If precaution costs  $x_J$  and  $x_K$  maximizing the objective functions under regimes  $J$  and  $K$  are generally unique, and if  $x_J \leq x_K$  holds, then the regime  $K$  provides more intensive precaution incentives than the regime  $J$ , in an obvious sense. Given the few assumptions imposed, however, such uniqueness cannot be taken for granted. To continue to rank incentives, I follow Milgrom and Shannon by making use of the strong set order. Accordingly, regime  $K$  is said to provide *more intensive precaution incentives* than regime  $J$  at standard  $x^o$  – for short,  $M(J, x^o) \leq_s M(K, x^o)$  – if, for any  $x_J \in M(J, x^o)$  and  $x_K \in M(K, x^o)$ , it follows that

$$x_J \wedge x_K = \min[x_J, x_K] \in M(J, x^o) \quad \text{as well as} \quad x_J \vee x_K = \max[x_J, x_K] \in M(K, x^o)$$

also hold. Notice that the binary operators  $\wedge$  and  $\vee$  impose a lattice structure on the set  $X$  of precaution spendings that are undominated in the sense of the previous section.

Suppose the intensity relation  $M(J, x^o) \leq_s M(K, x^o)$  holds and  $x_J$  and  $x_K$  are maximizers as above. If  $x_J \leq x_K$ , then, under regime  $K$ , more is actually spent on precaution than under regime  $J$ . If, however,  $x_K < x_J$ , then  $x_K = x_J \wedge x_K \in M(J, x^o)$  and  $x_J = x_J \vee x_K \in M(K, x^o)$  will both hold as well. Therefore, under regime  $J$ , the injurer would be equally well off by choosing  $x_J \wedge x_K$ , and under regime  $K$  he would be equally well off by choosing  $x_J \vee x_K$ . As a consequence, the injurer would raise no objection against a selection of maximizers where he spends more on precaution under regime  $K$  than under regime  $J$ .

The *intensity principle* as summarized by the following proposition provides a simple criterion for the intensity relationship to hold. The intensity principle is a special case of Theorem 4 in Milgrom and Shannon (1994). Proving the proposition directly, however, turns out to be less demanding than showing in detail why it is a special case of Theorem 4.

**PROPOSITION 1** *If the difference of the injurer's objective functions,  $\phi_K(x, x^o) - \phi_J(x, x^o)$ , is monotonically increasing in  $x$  from the range  $I(x^o)$ , then the intensity relation  $M(J, x^o) \leq_s M(K, x^o)$  holds.*

**PROOF** Suppose  $x_J \in M(J, x^o)$  and  $x_K \in M(K, x^o)$ . By definition, it follows that  $x_J \wedge x_K \leq x_J$ , and, since the difference of the objective functions is monotonically increasing, it follows that

$$\phi_K(x_J \wedge x_K, x^o) - \phi_J(x_J \wedge x_K, x^o) \leq \phi_K(x_J, x^o) - \phi_J(x_J, x^o)$$

must hold.

Notice further that either  $x_J \wedge x_K = x_J$  and  $x_J \vee x_K = x_K$  or  $x_J \wedge x_K = x_K$  and  $x_J \vee x_K = x_J$  must be true. In either case, it follows that

$$\phi_K(x_J \wedge x_K, x^o) - \phi_K(x_J, x^o) = \phi_K(x_K, x^o) - \phi_K(x_J \vee x_K, x^o) \geq 0,$$



where the nonnegativity is due to the fact that  $x_K$  maximizes  $\phi_K$ .

Combining the above inequalities immediately leads to

$$\phi_J(x_J, x^o) \leq \phi_J(x_J \wedge x_K, x^o) - [\phi_K(x_J \wedge x_K, x^o) - \phi_K(x_J, x^o)]$$

and hence to  $\phi_J(x_J, x^o) \leq \phi_J(x_J \wedge x_K, x^o)$ . Since  $x_J$  maximizes  $\phi_J$ , this inequality cannot hold in the strict sense, and  $x_J \wedge x_K$  must maximize  $\phi_J$  as well.

The remaining claim that  $x_J \vee x_K$  maximizes  $\phi_K$  follows analogously. *Q.E.D.*

Notice that if one of the objective functions has a unique maximizer or if the difference of the objective functions is strongly monotonically increasing, then  $x_J \leq x_K$  necessarily holds for any pair  $x_J$  and  $x_K$  of optimizers.

For regimes  $J \in \{S, N, P, W\}$ , it is easy to verify that the following differences of objective functions are monotonically increasing in the range  $I(x^o)$ :

$$\phi_P(x, x^o) - \phi_S(x) = \max[\varepsilon(x^o) \cdot L - \varepsilon(x) \cdot (L - H), 0],$$

$$\phi_N(x, x^o) - \phi_S(x) = \begin{cases} 0 & \text{if } x < x^o, \\ \varepsilon(x^o) \cdot H & \text{if } x = x^o, \end{cases}$$

$$\phi_W(x) - \phi_S(x) = S - G - \varepsilon(x) \cdot (L - H),$$

$$\phi_W(x) - \phi_P(x, x^o) = S - G + \min[-\varepsilon(x^o) \cdot L, -\varepsilon(x) \cdot (L - H)].$$

The following proposition combines such monotonicity with the intensity principle.

**PROPOSITION 2** *For any given standard  $x^o \in X$ , the following claims are valid:*

- (i)  $M(S, x^o) \leq_s M(P, x^o) \leq_s M(W, x^o) \leq_s M(W, \infty)$  and  $M(S, x^o) \leq_s M(N, x^o)$ .
- (ii) If  $x^o \notin M(N, x^o)$  then  $M(N, x^o) = M(S, x^o)$ , whereas if  $x^o \in M(N, x^o)$  then  $M(W, x^o) \subset M(P, x^o)$ .

**PROOF** Except for  $M(W, x^o) \leq_s M(W, \infty)$ , claim (i) follows directly from the intensity principle and the monotonicity of the corresponding differences of objective functions. The relation  $M(W, x^o) \leq_s M(W, \infty)$  holds for the following reason. If the standard is high enough, then  $M(W, x^o)$  is even a subset of  $M(W, \infty)$ ; otherwise,  $x^o < x_W$  holds for all  $x_W \in M(W, \infty)$ . In both cases, the intensity relation  $M(W, x^o) \leq_s M(W, \infty)$  will obviously be met.

As for claim (ii), if  $x^o \notin M(N, x^o)$  then  $M(N, x^o) = M(S, x^o)$ , as the two objective functions are the same in the relevant range, whereas if  $x^o \in M(N, x^o)$  then

$$\phi_S(x) \leq \phi_N(x, x^o) \leq \phi_N(x^o, x^o) = \phi_P(x^o, x^o)$$

holds for all  $x \in I(x^o)$ . Therefore, since

$$\phi_P(x, x^o) = \max[G - S + \varepsilon(x^o) \cdot L + \phi_W(x), \phi_S(x)]$$

holds for  $x \in I(x^o)$ , any maximizer of  $G - S + \varepsilon(x^o) \cdot L + \phi_W(x)$  or, what is the same, of  $\phi_W(x)$  must also maximize  $\phi_P(x, x^o)$  in the range  $I(x^o)$ . Claim (ii) is established. *Q.E.D.*

While the intensity principle refers to the intensity of incentives, under additional assumptions on the shape of expected welfare as a function of precaution, the above

proposition also provides normative insights with respect to the comparison of the negligence rule and proportional liability. Proportional liability always generates (weakly) less intensive precaution incentives than what would be first best (confined to the range  $I(x^o)$ ). But either the negligence rule generates even less intensive precaution incentives than proportional liability, or else all welfare-maximizing solutions constrained to the interval  $I(x^o)$  maximize the injurer's payoff function under proportional liability. Loosely speaking, proportional liability leads closer to the welfare optimum constrained to the range  $I(x^o)$  than the negligence rule. If, as traditionally assumed, social welfare is a concave function of the care level (single-peakedness as the purely ordinal counterpart would be enough), then incentives leading closer to the welfare optimum are always welfare-enhancing. Under such circumstances, proportional liability is superior in efficiency to the negligence rule at any common standard  $x^o$ .

A similar result has been established by Stremitzer and Tabbach (2009) for the traditional frame where the set  $X$  of precaution measures coincides with the set of nonnegative real numbers and where the probability of an accident is a differentiable and convex function of precaution expenditures. In contrast, the above proof makes use of ordinal properties only.

So far, the cap  $H$  on liability has been assumed constant. Under a plausible assumption, all results of the present section can easily be extended to a monotonically decreasing cap  $H = H(x)$ . The additional assumption can be expressed in terms of the victim's expected payoff function

$$\psi_S(x) = \varepsilon(x) \cdot [H(x) - L]$$

under strict liability, which is assumed (weakly) monotonically increasing with precaution, that is,  $\psi_S(x) \leq \psi_S(x')$  holds for  $x < x'$ . Under this assumption, all the above proofs remain valid. In fact, replacing  $H$  by  $H(x)$  and  $L - H$  by  $L - H(x)$  does not affect the monotonicity of differences of objective functions, and hence, by the intensity principle, intensity relations remain the same.

Beard (1990) has pointed out that the precaution incentives may possibly be counterintuitive if precaution expenditures lower the injurer's ability to compensate the victim. Yet, if the victim's payoff under strict liability remains weakly increasing with precaution spending, intensity relations remain qualitatively the same.

#### 4 Vicarious Liability under Moral Hazard

From now on, the injurer is an agent who takes precaution on behalf of his principal. To compensate for the wealth constraint of her agent, the principal may be held vicariously liable. Due to the combined wealth of principal and agent, under vicarious liability, the cap rises from  $H$  to  $H^v$  where  $0 \leq H \leq H^v \leq L$ . For the rest of the paper, these caps are assumed constant, but see the comment at the end of the previous section on possible extensions to nonconstant caps.

Much of the literature on vicarious liability assumes that the principal has full control over the agent's precaution decision at zero cost. If, however, their rela-

relationship operates under moral hazard, the principal may have to control the agent's precaution choice indirectly by offering a suitable bonus contract. The agent receives a fixed payment  $t$  plus the bonus  $b \geq 0$  if the accident is avoided.

Due to the agent's wealth constraint, the principal possibly has to pay a rent on top of the agent's outside option, as is well known from the principal-agent literature. Not surprisingly, this rent  $r(x)$  is a monotonically increasing function of the precaution  $x$  that the principal wants to implement. Moreover, depending on the exact structure of the underlying model, only a subset  $X^B$  of all levels can possibly be implemented by a bonus contract. Section A.1 of the appendix provides a formal proof of these statements, showing that no further assumptions are needed to establish monotonicity of the rent as a function of precaution spending.

It is a virtue of the intensity principle that, except for the monotonicity of the rent, details do not matter. In fact, since the rent is independent of the damages regime in place, it cancels whenever the difference of the principal's objective function under different regimes is examined to make use of the intensity principle. For that reason, all results of the previous section continue to hold as long as no comparison with first best is involved. But comparing with first best remains easy as well, because the only difference to the previous section concerns the rent, which is purely redistributive and hence does not affect welfare. Yet, since the rent is monotonically increasing, the intensity principle allows us immediately to rank precaution incentives relative to first best.

For the sake of completeness, I present a formal account of these extensions, even though the intuition gained in the previous section remains essentially the same. Suppose regime  $J$  is in place at standard  $x^o$ . Then the principal's objective function under vicarious liability  $v$  amounts to

$$\phi_J^v(x, x^o) = G - u - x - \varepsilon(x) \cdot \min[D_J(x, x^o), H^v] - r(x).$$

In fact, in order to induce the agent to take the job, the principal must cover the agent's outside option  $u$  and, in addition, must compensate the agent for his precaution effort, on top of which comes the rent required to induce  $x$ . Damages claims are borne by the principal as well. The principal's choice is confined to the set  $X^B$  of precautions implementable with a bonus contract.

To avoid tedious subcases, let me assume that courts impose implementable standards only, that is,  $x^o \in X^B$ . Then, under the negligence rule as well as under proportional liability, the principal will implement precaution from the range  $I^B(x^o) = \{x \in X^B : x \leq x^o\}$  only. For regimes  $J \in \{N, P\}$ , let

$$M^B(J, x^o) = \arg \max_{x \in I^B(x^o)} \phi_J^v(x, x^o)$$

be the set of precaution costs that maximize the corresponding objective function.

The information rent constitutes mere redistribution and, as such, does not affect welfare. For analytical convenience, however, it proves useful to include the additional regime  $Wr$  with objective function

$$\phi_{Wr}(x) = G^s - x - \varepsilon(x) \cdot L - r(x),$$

where the rent is deducted from welfare. Notice that the difference between welfare and the above objective function,

$$\phi_W(x) - \phi_{Wr}(x) = r(x),$$

is monotonically increasing in the whole range  $X$ .

Under regimes  $J \in \{S, Wr, W\}$  and for the same reason as in the previous section, consider the set

$$M^B(J, x^o) = \arg \max_{x \in I^B(x^o)} \phi_J^v(x)$$

of precaution costs that maximize the corresponding objective function,<sup>1</sup> but artificially confined to the range  $I^B(x^o)$ . In the absence of this constraint, optimal choices would be from  $M^B(J, \infty)$ .

The following proposition summarizes the adaptation needed to cover vicarious liability under moral hazard.

**PROPOSITION 3** *For any given standard  $x^o \in X^B$ , the following claims are valid:*

- (i)  $M^B(S, x^o) \leq_s M^B(P, x^o) \leq_s M^B(Wr, x^o) \leq_s M^B(W, x^o) \leq_s M^B(W, \infty)$  and  $M^v(S, x^o) \leq_s M^v(N, x^o)$ .
- (ii) *If  $x^o \notin M^B(N, x^o)$  then  $M^B(N, x^o) = M^B(S, x^o)$ , whereas if  $x^o \in M^B(N, x^o)$  then  $M^B(Wr, x^o) \subset M^B(P, x^o)$ .*

The above proposition compares precaution incentives under the different regimes for a setting of vicarious liability involving moral hazard. The results look very similar to the ones where the injurer was acting on his own. The superiority of proportional liability, however, is no longer supported. In fact, due to the information rent, precaution incentives under proportional liability are generally insufficient. As a consequence, the potentially more intensive precaution incentives under the negligence rule may lead closer to first best. In such cases, the negligence rule may well outperform proportional liability.

More generally, it can be shown that any precaution chosen under strict liability as well as any precaution chosen under proportional liability would also be chosen under the negligence rule, though possibly at a different standard. The following proposition establishes this claim formally.

**PROPOSITION 4** (i) *If  $x_S \in M^B(S, \infty)$  then  $x_S \in M^B(N, x_S)$ . (ii) If  $x_P \in M^B(P, x^o)$  then  $x_P \in M^B(N, x_P)$ .*

**PROOF** (i) If  $x_S \in M^B(S, \infty)$  then  $x_S \in M^B(S, x_S)$ . Moreover, if  $x < x_S$  then  $\phi_N^v(x, x_S) = \phi_S^v(x) \leq \phi_S^v(x_S) \leq \phi_N^v(x_S, x_S)$  and hence  $x_S \in M^B(N, x_S)$ . Claim (i) is established.

(ii) If  $x_P \in M^B(P, x^o)$  then

$$\phi_S^v(x) \leq \phi_P^v(x, x^o) \leq \phi_P^v(x_P, x^o) \leq \phi_P^v(x_P, x_P) = \phi_N^v(x_P, x_P)$$

for all  $x \in I^B(x^o)$  and hence  $x_P \in M^B(N, x_P)$ , as was to be shown. *Q.E.D.*

<sup>1</sup> To simplify notation, I use  $\phi_W^v = \phi_W$  and  $\phi_{Wr}^v = \phi_{Wr}$  even though these functions are independent of the cap under vicarious liability  $v$ .

Notice that a similar proposition could have been established for the setting of the previous section with an injurer acting on his own. In any case, if the standard of care is adapted to the damage regime in an appropriate way, the negligence rule outperforms all the other rules. Only at one and the same standard is it possible that proportional liability outperforms the negligence rule.

### 5 Precaution under Adverse Selection

If the relationship between a principal and an injurer (her agent) is characterized by adverse selection, the principal must be prepared to pay an information rent on top of the agent's outside option as well.

The principal expects the agent she faces to be of type  $i = 1, \dots, n$  with probability  $f_i$ , where  $f_1 + \dots + f_n = 1$ . The agent knows his type. If an agent of type  $i$  chooses precaution  $x \in X$ , his effort costs are type-contingent, amounting to  $c_i(x)$ . The set  $X$  is still understood as a (finite) subset of real numbers. Yet, since precaution costs are now type-contingent, elements of  $X$  will be referred to as precaution levels. Except for effort costs, the parameters of the model do not depend on type.

The principal may now wish to implement some type-contingent precaution profile  $y = (x_1, \dots, x_n)$  with the intention that an agent of type  $i$  should choose precaution  $x_i$  at subjective costs  $c_i(x_i)$  and receive payment  $t_i$  for it. Given adverse selection, the principal cannot directly identify the type of his agent, but instead must ensure that an agent knowing himself to be of type  $i$  has the incentive to choose precaution  $x_i$ .

Let  $Y = X \times \dots \times X$  denote the set of all precaution profiles. Not all of them will be implementable. Under the appropriate single-crossing property (for further details, the reader is referred to section A.2 of the appendix), it is the subset  $Y^m$  of monotonic profiles  $x_1 \leq \dots \leq x_i \leq \dots \leq x_n$  that can be implemented, a well-known result from mechanism design.

To implement the profile  $y \in Y^m$ , the principal must pay an expected rent  $r(y) = \sum_{i=1}^n f_i \cdot \rho_i$ , where  $\rho_i = t_i - c_i(x_i) - u \geq 0$  denotes the agent's rent on top of his outside option  $u$  if he is of type  $i$ . Moreover, it follows from the single-crossing property (see appendix A.2) that the expected rent is additively separable and can be written as

$$r(y) = \sum_{i=1}^n f_i \cdot r_i(x_i),$$

where the  $i$ th component is a monotonically increasing function  $r_i(x_i)$  of the  $i$ th agent's precaution  $x_i$ .

Given regime  $J$ , the principal's objective function  $\Phi_J(y, y^o)$  depends on the precaution profile  $y$  he wishes to implement and on the standards  $y^o = (x_1^o, \dots, x_n^o)$  of due care, which may or may not be type-contingent. To avoid lengthy subcases, however, I assume that these due-care standards are implementable, i.e.,  $y^o \in Y^m$ . Notice that noncontingent standards would always be implementable.

The objective function  $\Phi_J(y, y^o)$  of the principal inherits additive separability from the rent function and can be written as

$$\Phi_J(y, y^o) = \sum_{i=1}^n f_i \cdot \phi_{Ji}(x_i, x_i^o),$$

where  $\phi_{Ji}(x_i, x_i^o)$  is essentially of the same form as in the previous section. In fact,

$$\phi_{Ji}(x_i, x_i^o) = G - u - c_i(x_i) - \varepsilon(x_i) \cdot \min[D_J(x_i, x_i^o), H^v] - r_i(x_i),$$

where  $J$  denotes any of the regimes  $S, N,$  or  $P$ . Since the rent constitutes mere redistribution, it does not enter social welfare, i.e., for regime  $J = W$ , the components of the objective function are

$$\phi_{Wi}(x_i, x_i^o) = G^s - u - c_i(x_i) - \varepsilon(x_i) \cdot L.$$

As in the case of moral hazard, it proves analytically useful again to consider the additional regime  $J = Wr$  that deducts rents from welfare, i.e.,  $\phi_{Wri}(x_i, x_i^o) = \phi_{Wi}(x_i, x_i^o) - r_i(x_i)$ .

For the two regimes  $N$  and  $P$  involving a profile  $y^o = (x_1^o, \dots, x_n^o)$  of possibly type-contingent due-care standards, the principal cannot be better off by outperforming any of these standards, i.e., his optimal choice will be from the subset

$$I^m(y^o) = \{y \in Y^m : x_i \leq x_i^o \text{ for all } i\}$$

of all implementable profiles. For the other regimes, I impose this constraint under the same premises as in previous sections. Given regime  $J$ , profiles from the set

$$M^m(J, y^o) = \arg \max_{y \in I^m(y^o)} \Phi_J(y, y^o)$$

will be implemented that maximize the principal's expected payoff  $\Phi_J(y, y^o)$ .

To extend the intensity principle, it proves useful to endow the set  $Y^m$  of monotonic (i.e., implementable) precaution profiles with the following lattice structure. For any two precaution profiles  $y = (x_1, \dots, x_n)$  and  $y' = (x'_1, \dots, x'_n)$ , the precaution profiles  $y \wedge y'$  and  $y \vee y'$  are defined as those with the  $i$ th components

$$(y \wedge y')_i = x_i \wedge x'_i = \min[x_i, x'_i] \quad \text{and} \quad (y \vee y')_i = x_i \vee x'_i = \max[x_i, x'_i],$$

respectively. Notice that  $Y^m$  is indeed a lattice because, for any two monotonic profiles  $y$  and  $y'$ , both  $y \wedge y'$  and  $y \vee y'$  will be monotonic as well.

At profile  $y^o$  of standards, regime  $K$  is said to provide more intense precaution incentives than regime  $J$  if the strong set relationship  $M^m(J, y^o) \leq_s M^m(K, y^o)$  is met. The definition remains the same as in previous sections. It holds if and only if, for any pair  $y_J \in M^m(J, y^o)$  and  $y_K \in M^m(K, y^o)$  of maximizers, it follows that

$$y_J \wedge y_K \in M^m(J, y^o) \quad \text{and} \quad y_J \vee y_K \in M^m(K, y^o)$$

will hold as well. If this relationship is met, the principal would be willing to implement the precaution profile  $y_J \wedge y_K$  under regime  $J$  and profile under  $K$  such that each type of agent would spend (at least weakly) more under  $K$  than under  $J$ . In this sense, the intensity of precaution incentives increases type by type.

The following proposition adapts the intensity principle to the above setting of adverse selection.

PROPOSITION 5 *If, for all types  $i$  of agents, the difference of objective functions  $\phi_{K_i}(x_i, x_i^o) - \phi_{J_i}(x_i, x_i^o)$  is a monotonically increasing function of agent  $i$ 's precaution  $x_i$  (in the range  $x_i \leq x_i^o$ ), then the intensity relation  $M^m(J, y^o) \leq_s M^m(K, y^o)$  must hold.*

PROOF The proof makes use of the additive separability of the objective functions and parallels the one of Proposition 1.

In fact, suppose  $y_J = (x_{J_1}, \dots, x_{J_n}) \in M^m(J, y^o)$  and  $y_K = (x_{K_1}, \dots, x_{K_n}) \in M^m(K, y^o)$ . By definition, it follows that  $x_{J_i} \wedge x_{K_i} \leq x_{J_i}$  holds for all types. Since the differences of the objective functions are monotonically increasing type by type, it follows from additive separability that

$$\Phi_K(y_J \wedge y_K, y^o) - \Phi_J(y_J \wedge y_K, y^o) \leq \Phi_K(y_J, y^o) - \Phi_J(y_J, y^o)$$

must hold.

Notice further that for each type  $i$ , either  $x_{J_i} \wedge x_{K_i} = x_{J_i}$  and  $x_{J_i} \vee x_{K_i} = x_{K_i}$  or  $x_{J_i} \wedge x_{K_i} = x_{K_i}$  and  $x_{J_i} \vee x_{K_i} = x_{J_i}$  must be true. In either case, it follows again from additive separability that

$$\Phi_K(x_J \wedge x_K, x^o) - \Phi_K(x_J, x^o) = \Phi_K(x_K, x^o) - \Phi_K(x_J \vee x_K, x^o) \geq 0,$$

where the nonnegativity is due to the fact that  $x_K$  maximizes  $\Phi_K$ .

Combining the above terms immediately leads to

$$\Phi_J(x_J, x^o) \leq \Phi_J(x_J \wedge x_K, x^o) - [\Phi_K(x_J \wedge x_K, x^o) - \Phi_K(x_J, x^o)]$$

and hence to  $\Phi_J(x_J, x^o) \leq \Phi_J(x_J \wedge x_K, x^o)$ . Since  $x_J$  maximizes  $\Phi_J$ , this inequality cannot hold in the strict sense, and  $x_J \wedge x_K$  must maximize  $\Phi_J$  as well.

The remaining claim that  $y_J \vee y_K$  maximizes  $\Phi_K$  follows analogously. *Q.E.D.*

By comparing the above proof with the one of Proposition 1, it becomes obvious that the extension of the intensity principle to the present setting is based on exactly the same logic and intuition.

The same holds true for the monotonicity of differences. In fact, the differences

$$\phi_{K_i}(x_i, x_i^o) - \phi_{J_i}(x_i, x_i^o)$$

of objective functions remain monotonically increasing for the following pairs  $(J, K)$  of regimes:

$$(S, P), (P, Wr), (Wr, W), \text{ and } (S, N),$$

as follows from the corresponding findings of the previous sections. The final proposition is an immediate consequence of the intensity principle extended to the setting of adverse selection.

PROPOSITION 6 *For any profile  $y^o$  of standards, the following claims are valid:*

$$M^m(S, y^o) \leq_s M^m(P, y^o) \leq_s M^m(Wr, y^o) \leq_s M^m(W, y^o)$$

and

$$M^m(S, y^o) \leq_s M^m(N, y^o).$$

Except for the extension of the intensity principle as established in the previous proposition, no further proof is required to validate the above proposition, and hence no extra intuition is needed to support the result.

Demougin and Fluet have confined their comparison of strict liability versus the negligence rule to the case with the first-best profile as standard. In the second-best world of adverse selection, such standards need not be optimal. The above proposition applies to any monotonic profile of due-care standards, including noncontingent ones.

## 6 Concluding Remarks

This paper revisits the accident model at its roots. The injurer selects his precaution measure from a possibly finite set of alternatives. By eliminating dominated measures, the remaining set can be endowed with an order relationship. The intensity principle is then used as a unifying method to compare the intensity of precaution incentives under different legal regimes and under a variety of distortions due to inefficient due care levels, limited liability, wealth constraints, and information rents.

The proofs are both simpler and more general, as they do not rely on assumptions such as differentiability and convexity. No doubt, by imposing the more restrictive assumptions required for the first-order approach, similar results could also be established based on calculus. Yet, the results would be less general though harder to prove. The reader may try: take an accident model involving moral hazard and a possibly nonconstant cap on liability that remains insufficient to fully cover damages claims. Comparing the intensity of precaution incentives with calculus as method would certainly be a lengthy and painful exercise. The intensity principle, in contrast, can easily cope with such a setting.

On the legal side, the findings of the present paper are more difficult to digest. The economic analysis of tort law has argued in favor of due-care standards at their efficient level. Whether courts have followed such advice remains a matter of dispute. Suppose courts actually specify due-care standards in accordance with other criteria. In this case, the present paper still provides a positive theory of how the intensity of precaution incentives would be affected by inefficient standards.

Alternatively, suppose courts are actually aiming at raising welfare. If caps and information rents are involved, their task becomes quite complicated. The negligence rule awarding damages on an all-or-nothing basis may well be in line with court practice. The present paper has shown that the negligence rule offers a wide range of possible precaution incentives. Yet, increasing welfare would require some delicate fine tuning. To induce precaution beyond what can be achieved under proportional liability, the due-care standard would have to be raised to the point where it is still kept under the negligence rule. The efficient level (first best) could well be too high. Such fine tuning remains subtle, as the actual wealth constraints as well as the shape of the information rent would have to be taken into account, a difficult task indeed.



From an analytical perspective, however, the intensity principle has proven to be the appropriate method for handling even complicated versions of the accident model. As a matter of fact, the intensity principle provides a unifying setting that may be useful far beyond tort law. Quite generally, examining the monotonicity of the difference of objective functions proves a convenient approach if the intensity of incentives under different legal arrangements or relative to first best is at stake. In Schweizer (2012), I have examined the effects of breach remedies and performance excuses on investment decisions along similar lines. It seems promising to look for still further applications of the intensity principle in future research.

### Appendix

#### A.1 Moral Hazard

At a bonus contract with fixed payment  $t$  and bonus  $b \geq 0$ , the agent's objective function amounts to  $t + \psi(x, b)$  with  $\psi(x, b) = b \cdot (1 - \varepsilon(x)) - x$  such that he chooses precaution from the set

$$m(b) = \arg \max_{x \in X} t + \psi(x, b) = \arg \max_{x \in X} \psi(x, b),$$

which is independent of the fixed payment  $t$ . Precaution  $x \in X$  is implementable by a bonus contract (i.e.,  $x \in X^B$ ) if and only if there exists a bonus  $b$  such that  $x \in m(b)$ . To be feasible the fixed payment must satisfy two conditions,  $t + H \geq 0$  and  $t + \psi(x, b) \geq u$ . The first condition reflects the wealth constraint of the agent, and the second one is his participation constraint. In fact,  $t + \psi(x, b) - u \geq 0$  is the rent he receives on top of his outside option  $u$ .

For any given  $x \in m(b)$ , the principal goes for the lowest  $t$  that satisfies both conditions, viz.,  $t = \max[-H, u - \psi(x, b)]$ , which gives rise to rent

$$t + \psi(x, b) - u = \max[\psi(x, b) - u - H, 0].$$

Of all bonuses that implement  $x$  (if there are several), the principal offers the lowest one, denoted as  $b = \beta(x)$ , which can be determined as follows.

The bonus  $\beta$  implements precaution  $x$  if and only if, for all  $x' < x < x''$ , it holds that

$$\psi(x', \beta) \leq \psi(x, \beta) \quad \text{and} \quad \psi(x'', \beta) \leq \psi(x, \beta),$$

which is easily seen to be equivalent to

$$\frac{x - x'}{\varepsilon(x') - \varepsilon(x)} \leq \beta \leq \frac{x'' - x}{\varepsilon(x) - \varepsilon(x'')}.$$

Therefore, the lowest bonus  $\beta = \beta(x)$  that still implements  $x$  (if  $x$  is implementable at all) amounts to

$$\beta(x) = \sup_{x' < x} \frac{x - x'}{\varepsilon(x') - \varepsilon(x)}.$$

With this bonus, the rent  $r(x)$  is at its minimum and amounts to

$$r(x) = \max[\psi(x, \beta(x)) - u, 0].$$

To establish the monotonicity of this minimum rent, consider two precautions  $x_1 < x_2$  where  $x_i$  is implemented with bonus  $\beta_i$ , i.e.,  $x_i \in m(\beta_i)$ . Hence,

$$\psi(x_2, \beta_1) \leq \psi(x_1, \beta_1) \quad \text{and} \quad \psi(x_1, \beta_2) \leq \psi(x_2, \beta_2)$$

must both hold and are equivalent to

$$\beta_1 \cdot [\varepsilon(x_1) - \varepsilon(x_2)] \leq x_2 - x_1 \leq \beta_2 \cdot [\varepsilon(x_1) - \varepsilon(x_2)],$$

from which

$$0 \leq [\beta_2 - \beta_1] \cdot [\varepsilon(x_1) - \varepsilon(x_2)],$$

and hence  $\beta_1 \leq \beta_2$  follows immediately. As a consequence,

$$\max[\psi(x, \beta_1) - u, 0] \leq \max[\psi(x, \beta_2) - u, 0],$$

and hence  $r(x_1) \leq r(x_2)$  must hold. This completes the proof that the rent is a monotonically increasing function of precaution.

Readers preferring the traditional first-order approach would now have to introduce sufficiently many assumptions on the primitives of the model to ensure that any  $x$  is implementable by a bonus contract and that the rent is a differentiable function of precaution. If rigorously done, this is an unpleasant task that, fortunately, can be dispensed with by relying on the intensity principle. On top of that, since no further assumptions are needed, the result is of greater generality.

## A.2 Adverse Selection

In the case of adverse selection, the intensity principle has been extended to the situation where information rents are additively separable and where exactly those precaution profiles can be implemented that are monotonic. For these properties to hold, the following three assumptions turn out to be sufficient.

ASSUMPTION A1 *If  $x \in X$  then  $c_{i+1}(x) \leq c_i(x)$  (for  $i = 1, \dots, n - 1$ ).*

ASSUMPTION A2 *If  $x, x' \in X$  and  $x < x'$  then  $c_{i+1}(x') - c_{i+1}(x) < c_i(x') - c_i(x)$  (for  $i = 1, \dots, n - 1$ ).*

ASSUMPTION A3 *If  $x, x' \in X$  and  $x < x'$  then  $\varepsilon(x) > \varepsilon(x')$  and  $c_i(x) < c_i(x')$  (for  $i = 1, \dots, n$ ).*

Assumptions A1 and A2 – types with a higher index have lower costs and lower marginal costs – correspond to the single-crossing property well known from mechanism design. Assumption A3 is the familiar assumption traditionally imposed on the accident model.

For the reader's convenience, I briefly sketch the argument why the above three assumptions are indeed sufficient. Most of it is well known from mechanism design.

Under the three assumptions imposed, it is sufficient to check the local incentive constraints. Moreover, to minimize the rents, the principal will choose payments such that the *downward* constraints

$$t_{i+1} - c_{i+1}(x_{i+1}) = t_i - c_{i+1}(x_i)$$

are binding, i.e., an agent of type  $i + 1$  would just be indifferent between his contract  $(t_{i+1}, x_{i+1})$  and the one chosen by an agent of the next lower type  $i$ . Due to the single-crossing property, all the other incentive constraints will then be fulfilled a fortiori.

Participation constraints must be obeyed as well. Again due to the single-crossing property, it is sufficient to ensure participation of the lowest type, from which all the other participation constraints will follow. Moreover, to minimize expected rent payments, the principal offers payments such that the participation constraint of the lowest type is binding, i.e.,

$$\rho_1 = t_1 - c_1(x_1) - u = 0$$

will hold (no rent at the bottom).

For  $i = 2, \dots, n$ , the payments  $t_i$  as well as the rents

$$\rho_i = t_i - c_i(x_i) - u$$

that must be granted to an agent of type  $i$  can be calculated recursively from the binding downward incentive constraints.

In fact,

$$\rho_2 = t_2 - c_2(x_2) - u = c_1(x_1) - c_2(x_1) \geq 0$$

is a function of  $x_1$  and  $x_2$ , and similarly for  $\rho_i$ , which is a function of  $x_1, \dots, x_i$ . The expected rent amounts to

$$r(y) = \sum_{i=1}^n f_i \cdot \rho_i(x_1, \dots, x_i).$$

Since  $\rho_i(x_1, \dots, x_i)$  is additively separable, by rearranging terms we find functions  $r_i(x_i)$  such that

$$r(y) = \sum_{i=1}^n f_i \cdot r_i(x_i)$$

holds. Moreover, by making use of the single-crossing property again, it can be shown that the functions  $r_i(x_i)$  must in fact be monotonically increasing.

Under the traditional approach, further restrictive assumptions on second-order derivatives of cost functions would be needed to ensure the convexity of the rent functions, which, in turn, would be needed for reliance on first-order conditions. The intensity principle, in contrast, does not need any further assumptions. Not even differentiability is required.

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